Optimization of a Continuous Through-Circulation Dryer

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A mathematical model of a through-circulation packed bed dryer is presented in this paper. The semitheoretical model treats the case of constant rate drying in a deep bed of granular solids under initially uniform drying conditions. Experimental data for drying in superheated steam, mixtures of superheated steam and air, and air alone showed good agreement with predictions of the model.

Based on the model, a static optimization method is developed for optimal operation of a continuous through-circulation dryer subject to the inequality constraints of maximum permissible air-horsepower per unit bed area and maximum permissible local final moisture content in the bed. A nonlinear capacity function is defined in terms of the independent drying variables and is maximized under the above nonlinear restraints.

Wilde and Beightler (1) have recently described an optimization technique, the differential algorithm, which seems particularly suited to the highly nonlinear problem of optimizing a through-circulation dryer. These dryers are designed for the handling of particulate solids such as granulations, extrusions, etc., in the form of a packed bed. The bed is supported on a perforated plate or screen which is conveyed through the dryer (Figure 1). An appropriate drying agent (hot air, superheated vapor, etc.) is circulated through the bed, usually by centrifugal fans. The dryer may be compartmentalized with each section having its own fan and capable of directing flow through the bed either upward or downward. The effective dryer length is made up by connecting one or more compartments in series. In the case of an existing dryer for which the thermodynamic state of the drying fluid is fixed, the drying time is a function of the fluid velocity, bed loading, and the particle characteristics.

The conveyor speed, which is established by the drying time in a given dryer, need not be considered in the analysis.

DRYING MODEL

For purposes of analysis, the through-circulation dryer can be treated as a packed bed operating under the following conditions: adiabatic drying throughout the bed, plug flow of the fluid, uniform radial temperature distribution in the fluid, uniform and constant particle temperature distribution in the bed, uniform bed voidage, constant gas-solid heat transfer coefficient, and constant fluid transport properties.

For the constant rate drying period, the fluid temperature and bed moisture content distribution equations are sufficient to describe the process. These may be obtained from solutions of the differential energy equation for the gas phase and the coupled differential mass balance equation for the solid phase.

Differential energy equation (gas phase)

$$\frac{d^2T}{dz^2} - \frac{U}{\epsilon E}\frac{dT}{dz} - \frac{h'}{\epsilon E \rho_g C_g}(T - T_s) = 0 \qquad (1)$$

Differential mass equation (solid phase)

$$\frac{dm}{dt} + \frac{h'}{\lambda \rho_s}(T - T_s) = 0 \qquad (2)$$

Subject to boundary conditions on moisture content and gas temperature, the solutions in dimensionless form are

Gas-phase temperature distribution

$$\theta = e^{\phi Z} \qquad (3)$$

Solid-phase moisture content distribution

$$M = 1 - H\beta\Phi e^{\phi Z} \qquad (4)$$

The volumetric heat transfer coefficient for the bed is

$$h' = a_0\left(\frac{\rho_g U}{D_p}\right)^n \qquad (5)$$

For a given value of Z, Equation (4) describes the local drying curve.

When axial mixing can be considered negligible, Equations (3) and (4) reduce to

$$\theta = e^{-\beta Z} \qquad (6)$$

and

$$M = 1 - H\beta\Phi e^{-\beta Z} \qquad (7)$$

In formulating the optimization problem, it is useful to define an average dimensionless moisture content over the bed depth. Thus

$$\overline{M} = \int_0^1 M dZ = 1 - \frac{H\beta\Phi}{\phi}(e^{\phi} - 1) \qquad (8)$$

and, when axial mixing is ignored

$$\overline{M} = 1 + H\Phi(e^{-\beta} - 1) \qquad (9)$$

OPTIMIZATION PROBLEM

In formulating the optimization problem, we consider a through-circulation dryer of fixed geometry with flow directed through the bed by fans. Particle size is uniform. We choose as the objective function the production rate C, expressed as pounds of dry solids output per unit area of conveyor per unit time. Thus

$$C \equiv \rho_s L / t_F \qquad (10)$$

The production rate is not only a function of the defining variables of bed density, bed depth, and nominal residence time, but it can also be expressed in terms of the independent variables U, L, and D_p. We can get the specific form of the functionality by combining the definition above with the equation for the average dimensionless moisture content, Equation (9). The resulting equation does not contain the drying time as a variable:

$$C = \frac{\rho_s U H}{1 - \overline{M}_F} (1 - e^{-\beta}) \tag{11}$$

Equation (11) is the objective function to be maximized, subject to constraints imposed.

To avoid overloading the motors for the circulating blowers, we specify the maximum allowable power consumption per unit area of bed. The power consumed is directly proportional to the fluid velocity and to the corresponding pressure drop through the bed. By assuming that the pressure drop can be predicted by the Burke-Plummer equation (3), namely

$$\frac{\Delta P}{L} = \frac{1.75 \rho_g U^2 (1 - \epsilon)}{D_p \epsilon^3} \tag{12}$$

the power loss equation can be written as

$$P = \frac{1.75\ L U^3\ \rho_g (1 - \epsilon)}{g_c\ D_p \epsilon^3} \tag{13}$$

It should be noted that the applicability of the Burke-Plummer equation is restricted to a modified Reynold's number greater than 1,000; that is

$$N'_{Re} = \frac{D_p\ \rho_g\ U}{\mu_g\ (1 - \epsilon)} > 1{,}000$$

For most systems of interest, this limitation is met.

In stating the product specifications, we insist that the moisture content distribution in the bed as it leaves the dryer must be within a specified moisture range. If the direction of the fluid flowing through the bed is downward, then the minimum and maximum moisture contents will occur at the top and at the bottom of the bed, respectively. This is illustrated in Figure 2. Designating the minimum permissible moisture content as $M^0{}_F$ and the maximum as $M^L{}_F$, we write

$$M^0{}_F \leq (M_F)_{Z=0}$$

and

$$M^L{}_F \geq (M_F)_{Z=1}$$

Combining Equations (7) and (9) to eliminate Φ, we obtain

$$M = 1 - \beta \left[\frac{1 - \overline{M}}{1 - e^{-\beta}} \right] e^{-\beta Z} \tag{14}$$

When $t = t_F$ and $Z = 0$, Equation (14) becomes

$$(M_F)_{Z=0} = 1 - \beta \left[\frac{1 - \overline{M}_F}{1 - e^{-\beta}} \right] \geq M^0{}_F \tag{15}$$

At the other extreme of $t = t_F$ and $Z = 1$, Equation (14) is

$$(M_F)_{Z=1} = 1 - \beta \left[\frac{1 - \overline{M}_F}{e^{\beta} - 1} \right] \leq M^L{}_F \tag{16}$$

Rearranging Equation (15), we get

$$1 - M^0{}_F - f(\beta)\ e^{\beta} \geq 0$$

where

$$f(\beta) = \frac{\beta(1 - \overline{M}_F)}{e^{\beta} - 1}$$

or

$$\frac{1 - M^0{}_F}{f(\beta)} \geq e^{\beta}$$

But, from Equation (16)

$$f(\beta) \geq 1 - M^L{}_F$$

Therefore

$$\frac{1 - M^0{}_F}{1 - M^L{}_F} \geq e^{\beta} \tag{17}$$

The objective function and its associated constraints may be summarized as follows:

The objective function

$$C = A_2 U\ [1 - \exp\ (- A_1 L D_p{}^{-n} U^{n-1})] \tag{18}$$

The power constraint

$$P - K_1 \frac{L U^3}{D_p} \geq 0 \tag{19}$$

The moisture content constraint

$$\frac{1 - M^0{}_F}{1 - M^L{}_F} - e^{\beta} \geq 0 \tag{20}$$

The particle Reynolds number constraint

$$D_p U K_2 - 1{,}000 \geq 0 \tag{21}$$

where

$$A_1 = a_0 \rho_g{}^{n-1} C_g{}^{-1}$$

$$A_2 = C_g \rho_g (T_0 - T_s)/\lambda m_0 (1 - \overline{M}_F)$$

$$K_1 = \frac{1.75 \rho_g (1 - \epsilon)}{\epsilon^3}$$

$$K_2 = \rho_g / \mu_g (1 - \epsilon)$$

METHOD OF SOLUTION

The optimization problem we have posed is nonlinear both in the objective function and in the inequality constraints. However, the objective function is unimodal; consequently, the optimal solution will lie on the boundary of the feasible region. Before we begin the optimization program, it is convenient to transform the problem from one of maximization to one of minimization by changing the sign of the objective function. At the same time we shall adopt a simple nomenclature:

$$Y = A_2 \chi_1\ [\exp(- A_1 \chi_1{}^{n-1}\ \chi_2\ \chi_3{}^{-n}) - 1] \tag{22}$$

where

$$Y \equiv - C$$

$$\chi_1 \equiv U$$

$$\chi_2 \equiv L$$

$$\chi_3 \equiv D_p$$

The constraints become

$$f_1 = P - K_1\ \chi_1{}^3\ \chi_2\ \chi_3{}^{-1} \geq 0 \tag{23}$$

$$f_2 = \frac{1 - M^0{}_F}{1 - M^L{}_F} - \exp(A_1 \chi_1{}^{0.3}\ \chi_2\ \chi_3{}^{-1.3}) \geq 0 \tag{24}$$

$$f_3 = \chi_1\ \chi_3 - 1{,}000/K_2 \geq 0 \tag{25}$$

The necessary conditions for Y to be a minimum are given by the Kuhn-Tucker relations (4):

Nonnegativity of the constrained decision derivatives

$$\frac{\delta Y}{\delta d_p} \geq 0 \tag{26}$$

Nonnegativity of the constrained slack derivatives

$$\frac{\delta Y}{\delta f_t} \geq 0 \tag{27}$$

Complementary slackness conditions

$$\left(\frac{\delta Y}{\delta d_p}\right) d_p = 0 \qquad p - 1, \ldots, R \tag{28}$$

$$\left(\frac{\delta Y}{\delta f_t}\right) f_t = 0 \qquad t = 1, \ldots, M \tag{29}$$

where M is the number of tight constraints, N is the number of design variables, and $R = N - M =$ the number of decision variables. Sufficiency conditions are discussed by Wilde and Beightler (*1*).

As used here, the constrained derivatives measure the rate of change of the objective function with respect to a feasible change in a decision variable, all others held constant. In cases where $M \leq N$ (as it is here), the tight constraint variables f_t are treated as decision variables. The remaining M state variables are adjusted automatically by the algorithm to keep the value of the function on the boundary of the feasible region.

The algorithm, which depends on the evaluation of certain constrained derivatives, operates according to the following procedure.

1. Select any feasible, nonsingular, nondegenerate point and calculate the decision and slack derivatives by means of the Jacobian formulas:

$$\frac{\delta Y}{\delta d_p} = \frac{\partial(Y, f_1, \ldots, f_M)/\partial(d, s_1, \ldots, s_M)}{\partial(f_1, \ldots, f_M)/\partial(s_1, \ldots, s_M)} \tag{30}$$

$$\frac{\delta Y}{\delta f_t} = \frac{\partial(f_1, \ldots, f_{t-1}, Y, f_{t+1}, \ldots, f_M)/\partial(s_1, \ldots, s_M)}{\partial(f_1, \ldots, f_M)/\partial(s_1, \ldots, s_M)} \tag{31}$$

2. The constrained derivatives above are tested by the Kuhn-Tucker criteria.

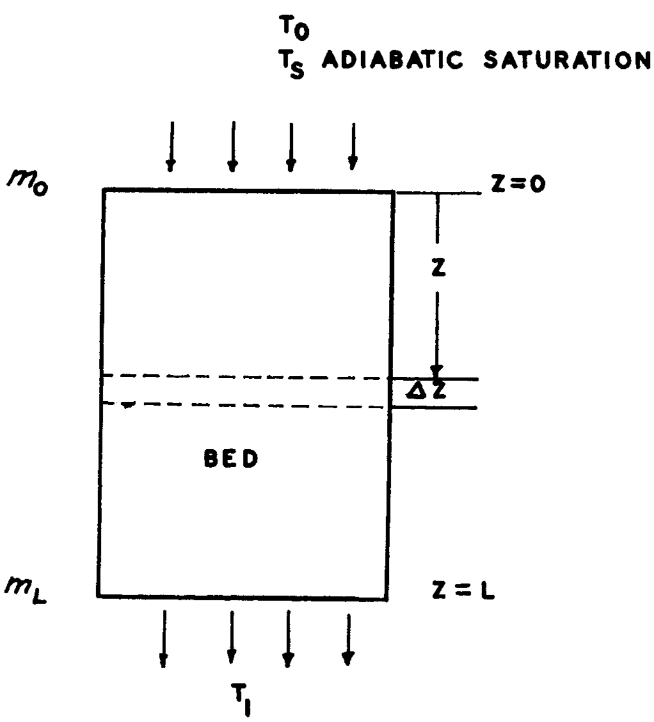

Fig. 1. Through-circulation dryer.

3. If any of the Kuhn-Tucker conditions are not satisfied, the direction to move for improvement is indicated by the values of the derivatives evaluated in step 1. Guidance in moving to the new, improved state point is also furnished by the loose constrained derivatives

$$\frac{\delta f_k}{\delta d_m} = \frac{\partial(f_k, f_1, \ldots, f_M)/\partial(d_m, s_1, \ldots, s_M)}{\partial(f_1, \ldots, f_M)/\partial(s_1, \ldots, s_M)} \tag{32}$$

$$\frac{\delta f_k}{\delta f_t} = \frac{\partial(f_1, \ldots, f_{t-1}, f_k, f_{t+1}, \ldots, f_M)/\partial(s_1, \ldots, s_M)}{\partial(f_1, \ldots, f_M)/\partial(s_1, \ldots, s_M)} \tag{33}$$

and by the state derivatives

$$\frac{\delta s_m}{\delta d_p} = - \frac{\partial(f_1, \ldots, f_M)/\partial(s_1, \ldots, s_{m-1}, d, s_{m+1}, \ldots, s_M)}{\partial(f_1, \ldots, f_M)/\partial(s_1, \ldots, s_M)} \tag{34}$$

$$\frac{\delta s_m}{\delta f_t} = \frac{\partial(f_{t+1}, \ldots, f_M, f_1, \ldots, f_{t-1})/\partial(s_{m+1}, \ldots, s_M, s_1, \ldots s_{m-1})}{\partial(f_1, \ldots, f_M)/\partial(s_1, \ldots, s_M)} \tag{35}$$

The sensitivity of the loose constraints and the state variables to perturbations of the decision variables are measured by the last four constrained derivatives. In Figure 3 the principal steps for analysis of a three variable problem are shown.

For purposes of numerical illustration, we have chosen to reduce the dimensionality by 1. Two factors are involved: the solution is easily presented in a step-by-step fashion, and we have experimentally verified data for a particular case. For our problem, the particle size is fixed, so we treat the bed depth and gas velocity as independent variables.

NUMERICAL EXAMPLE

Consider a through-circulation system for drying catalyst pellets. Feed moisture content of 0.55 lb. water/lb. of dry solid must be reduced to an average moisture content of 0.275 lb. water/lb. dry solid. The range of the moisture content distribution in the dried material is to be between 0.10 and 0.44 lb./lb. dry solid. Superheated steam at 320°F., and atmospheric pressure is to be the drying fluid.

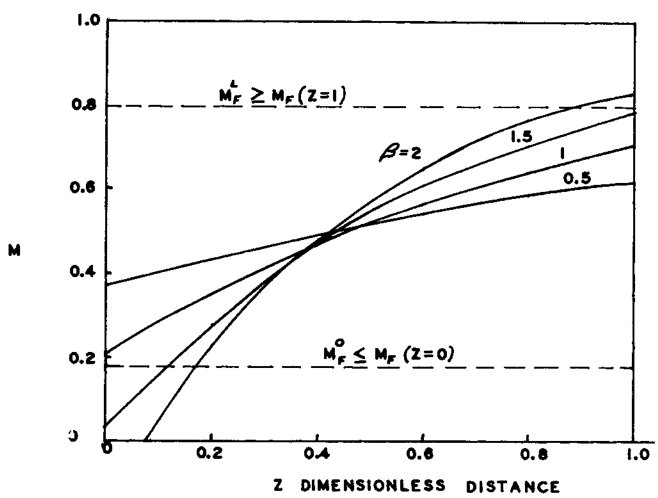

Fig. 2. Moisture content distribution at $\overline{M}_F = 0.50$.

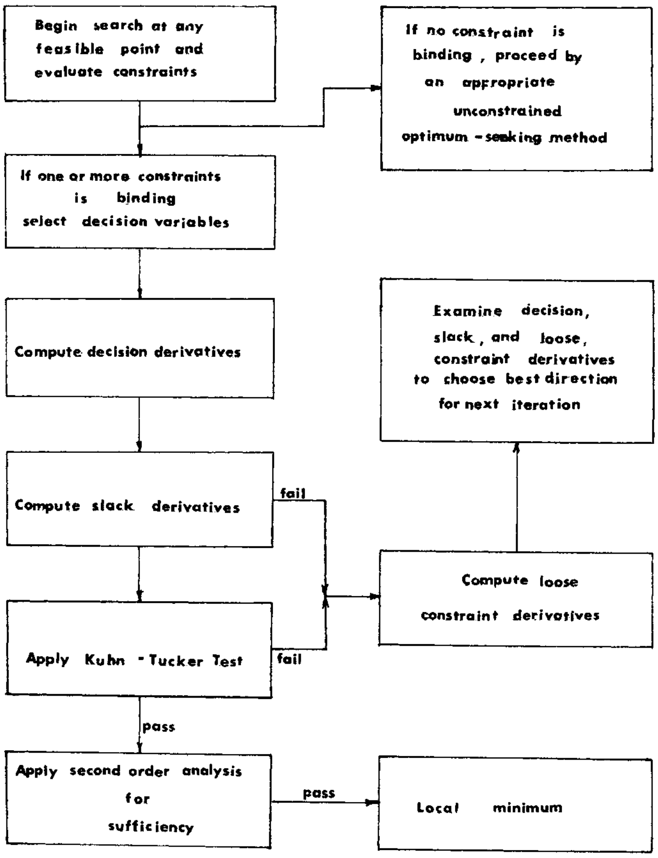

Fig. 3. Differential algorithm.

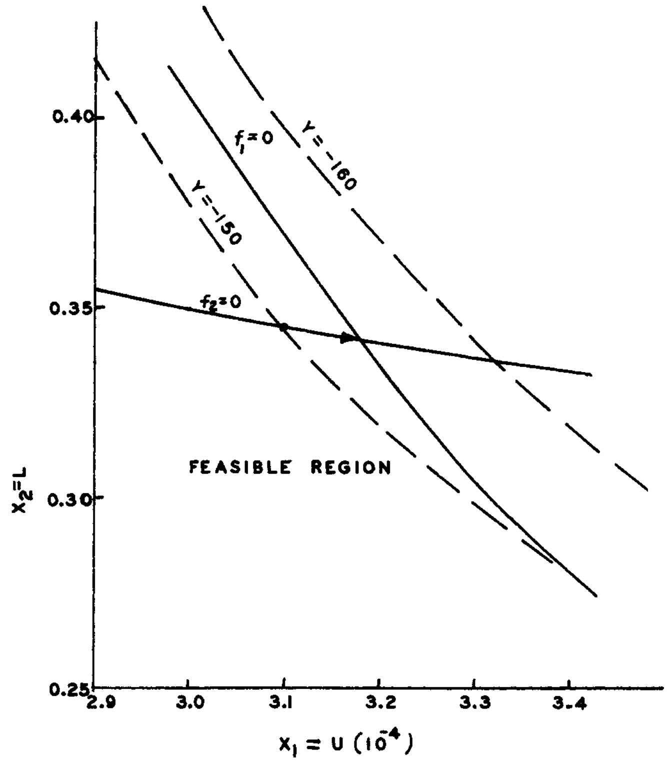

Fig. 4. Solution path.

In the existing unit, the power consumption of the blowers cannot exceed 0.156 hp./sq.ft. bed area. The effective diameter of the particles is fixed at a value of 0.375 in.

We are to estimate the magnitude of the fluid velocity U and the bed depth L which will maximize the production rate.

The design specifications and operating conditions for the dryer are summarized below:

$D_p = 0.375$ in.
$\overline{M}_F = 0.50$
$M^L{}_F = 0.80$
$M^0{}_F = 0.18$
$A_1 = 0.184$
$A_2 = 0.0064$
$P = 13\ (10^{13})\ \dfrac{(\text{lb.}_m)\ (\text{sq.ft.})}{(\text{hr.})^3}$ / sq.ft. of bed
$h' = 0.26(\rho_g U)^{1.3}$ B.t.u./(hr.) (°F.) (cu.ft. of bed)
$\epsilon = 0.45$

As an alternate to the Burke-Plummer equation, the pressure drop can be estimated by the Ergun pressure drop formula (5):

$$\Delta p = \frac{U(1-\epsilon)L}{D_p\ \epsilon^3}\left(\frac{150(\mu_g)(1-\epsilon)}{D_p} + 1.75\ \rho_g\ U\right) \tag{36}$$

The power loss equation becomes

$$P = (150 + K_2\ U)K_3\ LU^2 \tag{37}$$

where

$$K_2 = \frac{1.75 D_p \rho_g}{\mu_g(1-\epsilon)} = 0.05$$

and

$$K_3 = \frac{(1-\epsilon)^2 \mu_g}{D_p{}^2\ \epsilon^3} = 216$$

Because of the greater generality of Equation (34), no restriction is imposed on the particle Reynolds number. The objective function $Y\ (= -C)$ where

$$Y = 0.0064\chi_1 \exp(-0.184\chi_1{}^{0.3}\ \chi_2) - 0.0064\chi_1 \tag{38}$$

is to be minimized subject to the power constraint

$$f_1 = 1.2(10^{13}) - (3{,}000 + \chi_1)\ \chi_1{}^2\ \chi_2 \geqq 0 \tag{39}$$

and to the moisture content distribution constraint

$$f_2 = 4.1 - \exp\ (0.184\chi_1{}^{0.3}\ \chi_2) \geqq 0 \tag{40}$$

Following the computational scheme outlined previously, we begin the search at any feasible point, for instance $\chi_1 = 31{,}000$, $\chi_2 = 0.345$. At this point, $f_1 > 0$ and $f_2 = 0$. Since f_2 is a tight constraint, $M = 1$. The number of decision variables $N - M$ is then $2 - 1 = 1$, where N is the number of design variables. Let $\chi_1 = d_1 =$ decision variable. Then $\chi_2 = s_1 =$ state variable. In order to test a point for optimality, we must evaluate the following partial derivatives, which are used in checking for nonnegativity and complementary slackness:

$$\frac{\partial Y}{\partial d_1} = 0.0064\ [(\exp(-0.184\chi_1{}^{0.3}\ \chi_2))\ (1 - 0.0552\chi_1{}^{0.3}\ \chi_2) - 1]$$

$$\frac{\partial Y}{\partial s_1} = -\ 0.0064 - 0.00118\ \chi_1{}^{1.3} \exp(-0.184\chi_1{}^{0.3}\ \chi_2)$$

$$\frac{\partial f_1}{\partial d_1} = -\ [2\chi_1\chi_2(3{,}000 + \chi_1) + \chi_1{}^2\ \chi_2]$$

$$\frac{\partial f_1}{\partial s_1} = -\ (3{,}000 + \chi_1)\ \chi_1{}^2$$

$$\frac{\partial f_2}{\partial d_1} = -\ 0.0552\chi_1{}^{-0.7}\ \chi_2 \exp(0.184\chi_1{}^{0.3}\ \chi_2)$$

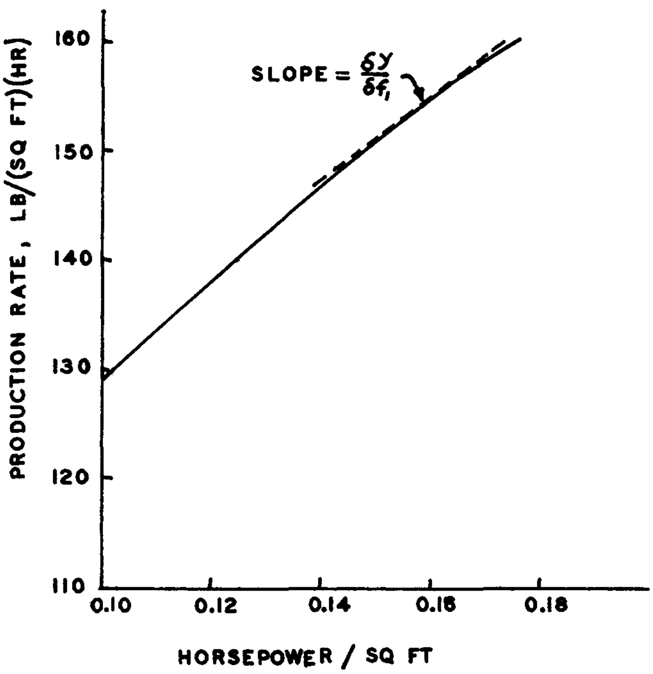

Fig. 5. Production rate vs. power.

$$\frac{\partial f_2}{\partial s_1} = -0.184\chi_1^{0.3} \exp(0.184\chi_1^{0.3}\chi_2)$$

Applying the nonnegativity test [Equation (26)] to the initial point, we get

$$\frac{\delta Y}{\delta d_1} = \frac{\partial(Y, f_2)/\partial(d_1, s_1)}{\partial f_2/\partial s_1} = \begin{vmatrix} \frac{\partial Y}{\partial d_1} & \frac{\partial Y}{\partial s_1} \\ \frac{\partial f_2}{\partial d_1} & \frac{\partial f_2}{\partial s_1} \end{vmatrix} \Bigg/ \frac{\partial f_2}{\partial s_1}$$

$$\frac{\delta Y}{\delta d_1} = -0.00417$$

The nonnegativity condition is violated, and Y consequently can be decreased by increasing d_1 keeping f_2 tight, that is, maintaining $f_2 = 0$. We investigate the effect of permitting f_2 to increase (loosen) as d_1 increases by computing the slack derivative $\delta Y/\delta f_2$:

$$\frac{\delta Y}{\delta f_2} = \frac{\partial Y/\partial s_1}{\partial f_2/\partial s_1} = 11.85$$

Clearly, allowing f_2 to increase, although permissible, does not improve Y. Thus, we are justified in keeping f_2 tight as d_1 is increased. To estimate how far to move along the constraint $f_2 = 0$, we compute

$$\frac{\delta f_1}{\delta d_1} = \frac{\partial(f_1, f_2)/\partial(d_1, s_1)}{\partial f_2/\partial s_1} = -9.58(10^8)$$

and

$$\frac{\delta s_1}{\delta d_1} = -\frac{\partial f_2/\partial d_1}{\partial f_2/\partial s_1} = -3.35(10^{-6})$$

Therefore, as d_1 (or χ_1) increases along $f_2 = 0$, the constraint f_1 tightens (approaches zero), and s_1 (or χ_2) decreases.

We continue to move along the second constraint until the first constraint ($f_1 = 0$) is encountered. Further movement along $f_2 = 0$ is prohibited. The location of the intersection is found by solving the system $\begin{cases} f_1 = 0 \\ f_2 = 0. \end{cases}$

The intersection is given by $\chi_1 = 31{,}800$ and $\chi_2 = 0.342$. At this point both constraints are tight; consequently, $M = 2$ and we have $R = 0$ (no degrees of freedom). Thus, $\chi_1 = s_1$ and $\chi_2 = s_2$; that is, both independent variables are state variables.

After computing the values of the unconstrained derivatives $\partial Y/\partial s_1$, $\partial Y/\partial s_2$, $\partial f_1/\partial s_1$, $\partial f_1/\partial s_2$, $\partial f_2/\partial s_1$, $\partial f_2/\partial s_2$ at the point (31,800, 0.342), we test the slack derivatives for nonnegativity:

$$\frac{\delta Y}{\delta f_1} = \frac{\partial(Y, f_2)/\partial(s_1, s_2)}{\partial(f_1, f_2)/\partial(s_1, s_2)} = +4.2(10^{-12})$$

and

$$\frac{\delta Y}{\delta f_2} = \frac{\partial(f_1, Y)/\partial(s_1, s_2)}{\partial(f_1, f_2)/\partial(s_1, s_2)} = +3.37$$

The complementary slackness conditions are also met, since

$$\left(\frac{\delta Y}{\delta f_1}\right)(f_1) = 0$$

and

$$\left(\frac{\delta Y}{\delta f_2}\right)(f_2) = 0$$

Wilde and Beightler (*1*) show that the above conditions are also sufficient.

The solution path is illustrated in Figure 4.

Accordingly, we have verified that the point ($\chi_1 = 31{,}800$, $\chi_2 = 0.342$) is a local minimum. Because the function is unimodal, this is also the global minimum. Consequently, the production rate C is maximum when a bed depth of 4.1 in. and a fluid velocity of 530 ft./min. are used.

It is important to note that at the optimum the derivatives $\delta Y/\delta f_1$ and $\delta Y/\delta f_2$ represent the sensitivity of the objective function to perturbations of the constraint expressions. For example, an increase of 0.01 hp./sq.ft. of bed area (constraint f_1) causes a 2.4% increase in the production rate. This is shown graphically in Figure 5. Similar analysis can be made for changes in the moisture content constraint f_2.

CONCLUSIONS

The method of optimization applied in this paper successfully minimizes a nonlinear objective function subject to nonlinear inequality constraints. The effects of perturbations in the decision variables on the objective function are routinely given by the method. Moreover, the use of the constrained derivative gives a measure of the sensitivity of the optimum solution to unpredicted deviations in the values of the constraint functions.

NOTATION

a_0, A_1, A_2 = numerical constants
C = production rate
C_g = gas-phase heat capacity
D_p = particle diameter
d_p = decision variable
E = total diffusivity
f_k = loose constraint variable
f_t = tight constraint variable
f_1 = power constraint function
f_2 = moisture distribution constraint function
f_3 = particle Reynolds number constraint function

H = thermal effectiveness factor = $\frac{C_g \rho_g (T_0 - T_s)}{\rho_s \lambda m_0}$
h' = volumetric heat transfer coefficient
L = bed depth
M = dimensionless local moisture content = m/m_0
$\overline{M}$ = dimensionless average moisture content
$\overline{M}_F$ = dimensionless final average moisture content
$M^0{}_F$ = dimensionless minimum permissible moisture content
$M^L{}_F$ = dimensionless maximum permissible moisture content
m_0 = initial moisture content
n = exponent
P = power consumption
s_m = state variable
T_0 = inlet gas temperature
T_s = adiabatic saturation temperature
t = time
t_F = time for passage of a unit mass of dry solid
U = superficial gas velocity
Z = dimensionless distance = z/L

Greek Letters

β = volumetric Stanton number = $h'L/U \rho_g C_g$
ϵ = bed void fraction
ϕ = dimensionless group = $\frac{U}{2\epsilon E}\left[1 - \left(1 + \frac{4h'\epsilon E}{C_g \rho_g U^2}\right)^{1/2}\right]$
Φ = dimensionless time = Ut/L
λ = latent heat of vaporization
ρ_g = gas-phase density
ρ_s = bulk density of bed
μ_g = gas-phase viscosity
θ = dimensionless temperature = $(T - T_s)/(T_0 - T_s)$

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Feedforward Computer Control of a Class of Distributed Parameter Processes

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A finite difference technique has been developed which is suitable for the derivation of feedforward control algorithms for the control of distributed parameter processes. The method embodies both steady state and transient feedforward compensation.

An experimental study, involving closed loop, on line, analogue computer control of a simple distributed parameter process proves the feasibility of the finite difference technique. The process under study is a steam water heat exchanger subject to inlet temperature forcing and controlled by flow rate manipulation. The new technique yielded significantly improved results when compared with the performance of a conventional two-mode feedback controller, or a linearized feedforward control algorithm.

The use of analogue and particularly digital computers within the past 15 yr. has projected theoretical developments in the process control field far ahead of the experimental. Simulation of equations has, to a large extent, displaced the laboratory verification of advanced control schemes.

The need for experimental evidence in this area is supported by the following arguments: mathematical models cannot exactly predict real process behavior, and the extent of the difference cannot be determined except by experiment; process noise can preclude or seriously limit the effectiveness of sophisticated control schemes such as Liapunov (*1, 2*) or model reference adaptive control (*3, 4*) which can require the differentiation of signals; real transducers, control elements (valves, etc.), will be of limited accuracy and may be subjected to nonlinearities such as sticking and hysteresis (*5*) which are not taken

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